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★**Lie-Cartan-Ehresmann theory.**

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In this book the author tries to elaborate in a systematic way much of the foundational material and many of the theories developed by Lie, Cartan, and Ehresmann, the three mathematicians who have been his inspiration since he was a graduate student. The theme of the book centers around Élie Cartan's magnificent ideas of the theory of geometric structures and its translation into a fiber-bundle-calculus-on-manifolds framework by Ehresmann (in this setting, a geometric structure on a finite-dimensional smooth manifold X may be considered as a collection of sub-bundles of one or more of the jet bundles $J^r(X, Y)$, $r = 1, 2, \cdots$, where Y is another finite-dimensional smooth manifold). From this the book radiates out to many disciplines such as moving coframes and Lie-Cartan pseudogroups, the method of exterior differential systems in the calculus of variations as first formulated by P. A. Griffiths [*Exterior differential systems and the calculus of variations*, Progr. Math., 25, Birkhäuser Boston, Boston, MA, 1983; MR0684663 (84h:58007)], Lie structures and the Kodaira-Spencer theory of deformations of geometric structures in work by S. Lie [Math. Ann. **25** (1885), 71–151; Jbuch **17**, 339; English translation in *Cartanian geometry, nonlinear waves, and control theory, Part B*, Math. Sci. Press, Brookline, MA, 1980; MR0564652 (81m:58010)], and so on.

While everyone would agree that Cartan's work is full of insight and ideas, that the special tricks and brilliant observations in his "examples" are themselves deep theories and in fact, many of his examples are topics in their own right, it is equally agreed upon that it takes one an unbelievably long period of time to understand and digest Cartan's ideas and methods, and it would definitely do younger mathematicians much good should his work be systematically written in the form of comprehensible theories. Although this would be a huge project, R. L. Bryant et al. [*Exterior differential systems*, Springer, New York, 1991; MR1083148 (92h:58007)] have already done a very remarkable job, and the present beautifully written volume is another excellent addition to the exploration of Cartan's methods. *Wing-Sum Cheung*